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Compact Kähler Manifolds with Parallel Ricci Tensor

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Introduction

Kähler manifolds with parallel Ricci tensor, which are called generalized Einsten Kähler manifolds in some literature, are locally direct products of Einstein Kähler manifolds. In this note we shall study global structures of a compact Kähler manifold locally decomposable as an isometric product of Ricci-positive, Ricci-flat and Ricci-negative parts:

Theorem. *Suppose that the universal covering space $\tilde{X}$ of a compact Kähler manifold X is (isometrically biholomorphic to) a Kählerian direct product $P \times F \times N$, where P, F, N are Kähler manifolds with positive, zero, negative Ricci curvature respectively. Then*

a) *X is a holomorphic fiber bundle $X \to Y$ with fiber P over a compact Kähler manifold Y, and*

b) *Y admits a holomorphic map $Y \to Z$ onto a compact Kähler V-manifold Z such that the universal covering space of a typical fiber is F.*

If P reduces to a point, namely $\tilde{X} \cong F \times N$, our theorem gives a partial affirmative answer to the Abundance Conjecture (see, e.g., a survey article [KMM]). For a compact Riemannian manifold with non-positive sectional curvature, an analogous result was obtained by Eberlein [E, Cor. 2].

The problem treated in this note can be regarded as an existence problem of compact leaves for the foliations on X and Y induced respectively by the projection $P \times F \times N \to F \times N$ and $F \times N \to N$. The compactness of leaves in P-directions, a) in Theorem, can be derived only from the positivity of Ricci curvature in these directions. On the other hand, the compactness of leaves in F-directions, b) in Theorem, depends on both tangential and transversal structures of the foliation. We illustrate this point with the following two examples.

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