Advanced Studies in Pure Mathematics 17, 1989
Algebraic Number Theory — in honor of K. Iwasawa
pp. 471–492

Duality Theorems for Abelian Varieties over Z_p-extensions

Kay Wingberg

Dedicated to Kenkichi Iwasawa on his 70th birthday

Our concern in this paper is to define p-adic height pairings for an abelian variety A over an algebraic number field k on the niveau of a Z_p-extension k_∞ of k. We will show that there exists a map from the Λ-torsion submodule $T_\Lambda H^1(\mathcal{O}_\infty, \mathscr{A}(p))^*$ of the Pontrjagin dual of the p-Selmer group to the adjoint α of the corresponding module for the dual abelian variety A'. Here Λ denotes the completed group ring of $\mathrm{Gal}(k_\infty/k)$ over Z_p and p is a prime number where A has good reduction. $\mathscr{A}$ denotes the Néron model defined over the ring of integers $\mathcal{O}_\infty$ of k_∞. More generally, for $i \geqq 0$ there are canonical maps

$$T_\Lambda H^i(\mathcal{O}_\infty, \mathscr{A}(p))^* \longrightarrow \alpha(T_\Lambda H^{2-i}(\mathcal{O}_\infty, \mathscr{A}'(p))^*).$$

These maps are quasi-isomorphisms if A has ordinary good reduction at p. In this case they can be regarded as non-degenerate pairings between the Λ-torsion submodules of $H^i(\mathcal{O}_\infty, \mathscr{A}(p))^*$ and of $H^{2-i}(\mathcal{O}_\infty, \mathscr{A}'(p))^*$. The pairing induced on a finite layer k_n/k coincides with the pairing defined by Schneider [8] (for $i=1$ and assuming that $H^1(\mathcal{O}_\infty, \mathscr{A}(p))^*$ is Λ-torsion and fulfills a certain semi-simplicity property).

Furthermore, we define an Iwasawa L-function in terms of characteristic polynomials of $T_\Lambda H^i(\mathcal{O}_\infty, \mathscr{A}(p))^*$:

$$L_p(A, \kappa, s) = \prod_{i=0}^{2} F_i(\kappa(\phi)^{s-1}-1)^{(-1)^{i+1}}, \qquad s \in Z_p,$$
$$F_i(t) = p^{\mu_i} \det(t-(\phi-1);\, T_\Lambda H^i(\mathcal{O}_\infty, \mathscr{A}(p))^* \otimes Q_p),$$

where κ is the character corresponding to k_∞, ϕ is a generator of $\mathrm{Gal}(k_\infty/k)$ and μ_i is the μ-invariant of $H^i(\mathcal{O}_\infty, \mathscr{A}(p))^*$. In the ordinary case the pairing mentioned above leads to a functional equation for $L_p(A, \kappa, s)$ with respect to $s \mapsto 2-s$. This generalizes a result of Schneider [8] and

Received July 1, 1987.