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On Sinnott's Proof of the Vanishing of the Iwasawa Invariant μ_p

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To my teacher, Professor Iwasawa, on his seventieth birthday

In [3], W. Sinnott gave a new proof of the result of B. Ferrero and the present author [1] that the Iwasawa invariant μ_p vanishes for cyclotomic $\mathbb{Z}_p$-extensions of abelian number fields. The original proof was based on Iwasawa's construction of p-adic L-functions [2] and also used the concept of p-adic normal numbers. Sinnott replaced the results on normal numbers with a purely algebraic independence result (Lemma 2 below), which enabled him to work in the context of p-adic measures and distributions and to prove that (approximately) the μ-invariant of a rational function equals the μ-invariant of its Γ-transform. In the present note, we show that Sinnott's proof can be translated back into the language of Iwasawa power series. It is amusing to note that the step involving the Γ-transform, while not very difficult to begin with, is now replaced by the even simpler observation that if a prime divides the coefficients of a polynomial then it still divides them after a permutation of the exponents.

We first introduce the standard notation (see [4, p. 386] for more details): p is a prime; $q=4$ if $p=2$ and $q=p$ if p is odd; χ is an odd Dirichlet character of conductor f, where f is assumed to be of the form d or qd with $(d,p)=1$ (i.e., χ is a character of the first kind); $q_n=dqp^n$; $i(a)=-\log_p(a)/\log_p(1+q_0)$ for $a\in\mathbb{Z}_p$, where $\log_p$ is the p-adic logarithm; $\mathcal{O}=\mathbb{Z}_p[\chi(1),\chi(2),\cdots]$; (π) is the prime of $\mathcal{O}$; $\Lambda=\mathcal{O}[\![T]\!]$; K=field of fractions of $\mathcal{O}$; α runs through the $\phi(q)$-th (2nd or $(p-1)$-st) roots of unity in $\mathbb{Z}_p$; $\langle a\rangle$ is defined for $a\in\mathbb{Z}_p^\times$ by $a=\omega(a)\langle a\rangle$, where ω is the Teichmüller character; $\{y\}$ is the fractional part of $y\in\mathbb{Q}$; $\omega_n(T)=(1+T)^{p^n}-1$; and

$$B(y)=(1+q_0)\{y\}-\{(1+q_0)y\}-\frac{q_0}{2}.$$

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