

Motivic Iwasawa Theory

Peter Schneider

Dedicated to K. Iwasawa

The aim of this paper is to convince the reader that there is a general theory of p -adic L -functions for varieties over number fields which to a large extent parallels the theory of complex L -functions. We will develop a cohomological formalism which in some sense can be viewed as a theory of Iwasawa theoretic realizations of motives. From these realizations we then construct the p -adic L -functions via Iwasawa's abstract notion of a characteristic power series for an Iwasawa module (in order to make this work we unfortunately need, in the moment, some restrictive assumptions on the variety). We will establish various basic properties of these L -functions the most important of which is that they have a functional equation. We also will begin to discuss some of their finer properties like the location and multiplicities of their zeros and poles. Since such a task requires a careful motivation and justification in each step we decide to expand the introduction into a paragraph of its own right giving here only the list of the contents:

1. Motivation
2. The two approaches
(A) Greenberg's approach (B) The $\Gamma(n)$ -approach
3. First results
4. The rank conjecture in the ordinary case
5. p -adic L -functions in the ordinary case
6. The functional equation
7. The comparison between the two approaches
8. Zeros, values, and p -adic regulators
9. Remarks on the nonordinary case

The paper is divided into two parts. This first part covers the paragraphs 1–5.