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Bimodules and Abelian Surfaces

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To Professor K. Iwasawa

Introduction

In a manuscript on mod ℓ representations attached to modular forms [26], the author introduced an exact sequence relating the mod p reduction of certain Shimura curves and the mod q reduction of corresponding classical modular curves. Here p and q are distinct primes. More precisely, fix a maximal order $\mathcal{O}$ in a quaternion algebra of discriminant pq over $\boldsymbol{Q}$. Let M be a positive integer prime to pq. Let $\mathscr{C}$ be the Shimura curve which classifies abelian surfaces with an action of $\mathcal{O}$, together with a "$\Gamma_o(M)$-structure." Let $\mathscr{X}$ be the standard modular curve $X_o(Mpq)$. These two curves are, by definition, coarse moduli schemes and are most familiar as curves over $\boldsymbol{Q}$ (see, for example, [28], Th. 9.6). However, they exist as schemes over $\boldsymbol{Z}$: see [4, 6] for $\mathscr{C}$ and [5, 13] for $\mathscr{X}$.

In particular, the reductions $\mathscr{C}_{\boldsymbol{F}_p}$ and $\mathscr{X}_{\boldsymbol{F}_q}$ of $\mathscr{C}$ and $\mathscr{X}$, in characteristics p and q respectively, are known to be complete curves whose only singular points are ordinary double points. In both cases, the sets of singular points may be calculated in terms of the arithmetic of "the" rational quaternion algebra which is ramified precisely at q and ∞. (There is one such quaternion algebra *up to isomorphism.*) In [26], the author observed that these calculations lead to the "same answer" and concluded that there is a 1–1 correspondence between the two sets of singular points. He went on to relate the arithmetic of the Jacobians of the two curves $\mathscr{X}$ and $\mathscr{C}$ (cf. [14] and [10, 11]).

The correspondence of [26] depends on several arbitrary choices. More precisely, [26] used Drinfeld's theorem [6] to view the Shimura curve $\mathscr{C}$ over $\boldsymbol{Z}_p$ as the quotient of the appropriate "p-adic upper half-plane" by a discrete subgroup Γ of $\boldsymbol{PGL}_2(\boldsymbol{Q}_p)$. This group is obtained by

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