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A Lower Bound of $L_p(1, \chi)$ for a Dirichlet Character χ

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Dedicated to Professor Kenkichi Iwasawa on his seventieth birthday

Let χ be a primitive Dirichlet character with conductor f, let $\tau(\chi)$ be the Gaussian sum for χ, and let $L_p(s, \chi)$ be the p-adic L-function associated with χ. Then, by the results of Brumer [3] and Leopoldt [8], the value $L_p(1, \chi)$ of this function at $s=1$ is not zero, and is given by the following formula:

$$L_p(1, \chi) = -\left(1 - \frac{\chi(p)}{p}\right)\frac{\tau(\chi)}{f}\sum_{a=1}^{f}\bar{\chi}(a)\log(1-\zeta^{-a}).$$

Since this value is related to the class numbers of cyclotomic fields, it is important to obtain a lower bound of $L_p(1, \chi)$.

Since the above formula expresses $L_p(1, \chi)$ in a linear form of p-adic logarithms of algebraic numbers, it is natural to study lower bounds of linear forms of p-adic logarithms of algebraic numbers by Baker's method. There are several results in this direction (cf. Spindzhuk [10], Kaufman [6], van der Poorten [9], *etc.*). But some results are not explicit enough for us, and some paper has (minor) mistakes so that the resulting constants must be modified (cf. Remark in 2–1). Since the values of the constants are essential for our purpose, we first study this problem. Then, calculating the relevant constants, we obtain a lower bound of $L_p(1, \chi)$.

In § 1, we improve a result of Gel'fond [4] on p-adic interpolations of p-adic normal functions by polynomials. In § 2, we calculate lower bounds of linear forms in p-adic logarithms of algebraic numbers by the method of Baker [2]. In § 3, we use the explicit formula of $L_p(1, \chi)$ and, by calculating the relevant constants, obtain a lower bound of $L(1, \chi)$.

The author first studied this problem by the method of Kaufman [6]. Then he heard the existence of van der Poorten [6] from M. Waldschmidt. So he used the method of Baker [2] and improved the lower bound. After writing this paper, the author met A. Baker and heard that Waldschmidt improved Baker's result in [11], and that a Chinese mathematician also

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