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Cyclotomic Z_p-extensions of $Q(\sqrt{-1})$ and $Q(\sqrt{-3})$

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Dedicated to Professor Kenkichi Iwasawa on his 70th birthday

In the theory of Z_p-extensions of a number field, the λ-invariant has a special meaning that it is an analogue of the genus of an algebraic curve. In this point of view, one can naturally hope that there exists a uniform bound for λ_p independent of p when the base field is fixed, and this bound might be regarded as the genuine analogue of the genus for a number field. This question has been studied by Ferrero [1, 2] and Metsänkylä [5, 6].

In this paper, we refine Ferrero's results for some imaginary quadratic fields, in particular for $Q(\sqrt{-1})$ and $Q(\sqrt{-3})$.

§ 1.

We describe briefly how to get the exact values of a p-adic measure α defined below. We follow Sinnott [7] to construct a p-adic L-function. Let θ be an odd Dirichlet character with conductor d. We assume d is not a power of p. Define a rational function for θ by

$$F_\theta(X)=\textstyle\sum_{a=1}^{d}\theta(a)(1+X)^a/\{(1+X)^d-1\}.$$

Let $\mathcal{O}$ be the integer ring of the field generated over Q_p by the values of θ, and let π be a prime element of $\mathcal{O}$. Then $F_\theta(X)$ can be expanded into a formal power series with $\mathcal{O}$-coefficients. Let α be the $\mathcal{O}$-valued p-adic measure corresponding to F_θ. Replace the period d in F_θ by dp^n. Then we get the following congruence from the fundamental correspondence between measures and power series:

$$\alpha(r+(p^n))(1+X)^r\equiv\{\textstyle\sum'\theta(a)(1+X)^a\}/\{(1+X)^{dp^n}-1\}$$
$$(\mathrm{mod}\ (1+X)^{p^n}-1),$$

where r is an integer satisfying $0\leq r<p^n$, and the sum $\sum'$ is taken over all integers a with $1\leq a<dp^n$, $a\equiv r\ (\mathrm{mod}\, p^n)$. Put $X=0$. Then we have

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