Advanced Studies in Pure Mathematics 17, 1989
Algebraic Number Theory — in honor of K. Iwasawa
pp. 139–169

On Nearly Ordinary Hecke Algebras for $GL(2)$ over Totally Real Fields

Haruzo Hida

Dedicated to Kenkichi Iwasawa for his 70th birthday

Since this work is a continuation of our previous paper [8], we shall suppose the familiarity on the reader's part with the result and the notation in [8]. Especially we fix a rational prime p and a totally real field F of degree d. Let $\mathcal{O}$ be a valuation ring finite flat over $\boldsymbol{Z}_p$ containing all the conjugates of the integer ring r of F, and let Z (resp. $\bar{Z}$) be the Galois group of the maximal abelian extension of F unramified outside p and ∞ (resp. outside p). We denote by W the torsion-free part of Z. Then, for each weight $v>0$, we have constructed in [8] the ordinary Hecke algebra finite and without torsion over the continuous group algebra $\mathcal{O}[\![W]\!]$. The significance of this algebra lies in the fact that for every non-negative weight n parallel to $-2v$ (i.e. the sum $n+2v$ is a parallel weight), the Hecke algebra $\mathfrak{h}^{\mathrm{ord}}_{k,w}(p^\alpha;\mathcal{O})$ $(k=n+2t,\ w=v+k-t)$ over $\mathcal{O}$, for the space of holomorphic Hilbert cusp forms for F of level p^α and of weight (k, w), can be uniquely obtained as a residue algebra of $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$. Strictly speaking, this algebra $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ cannot be called ordinary when $v>0$ since the Hecke operator $T(p)$ in $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ is in fact divisible exactly by p^v. This algebra $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ cannot be either said universal because of the restriction to the weight n which requires n to be parallel to the fixed weight $-2v$. Thus, there exists infinitely many Hecke algebras $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ parametrized by the weights v modulo parallel ones. In this paper, we shall unify these infinitely many Hecke algebras $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ and construct a unique universal one $\boldsymbol{h}^{\mathrm{ord}}(1;\mathcal{O})$ from which each $\boldsymbol{h}^{\mathrm{ord}}_v(1;\mathcal{O})$ can be obtained as a residue algebra for all non-negative v.

Although we shall postpone the exact formulation of our result to § 2 of this paper, let us make it a little more precise. Let $\bar{Z}_0$ be the subgroup of Z generated by the inertia groups at all the prime ideals of r over p, and put $G=\bar{Z}_0\times r_p^\times$, where $r_p=r\otimes_{\boldsymbol{Z}}\boldsymbol{Z}_p$ is the p-adic completion of the integer ring r of F. Let I be the set of all embeddings of F into $\bar{\boldsymbol{Q}}$, and put $t=\sum_\sigma \sigma \in \boldsymbol{Z}[I]$. We shall say that a weight $k \in \boldsymbol{Z}[I]$ is parallel if k

Received December 23, 1987.