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On the Uniqueness of Frobenius Operator on Differential Equations

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Dedicated to K. Iwasawa

Let k be an algebraically closed field of characteristic zero complete under an ultrametric norm with residue class field of characteristic p. Let $\mathscr{H}$, $\mathscr{G}$ be $d\times d$ matrices with coefficients in $k(x)$, the field of rational functions in one variable. We consider differential equations

$$\frac{dy}{dx}=y\mathscr{H} \tag{1}$$

$$\frac{dy}{dx}=y\mathscr{G} \tag{2}$$

with a "Frobenius operation" mapping solutions of (1) into solutions of (2), i.e., we assume the existence of a $d\times d$ matrix A with non-trivial determinant and coefficients analytic (in the sense of Krasner) on the complement of a finite number of disks each lying properly in a residue class such that for each solution matrix, Y, of (1) analytic in a region meeting the support of A, the product

$$Y(x^p)A=Y_2(x) \tag{3}$$

is a solution matrix of (2). This is equivalent to the differential relation

$$\frac{dA}{dX}=A\mathscr{G}-px^{p-1}\mathscr{H}(x^p)A. \tag{4}$$

Our object is to state conditions which imply the uniqueness of A up to a constant (scalar) factor.

Lemma. *Suppose*

(α) *Equation* (1) *has a solution matrix analytic on the generic open*

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