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p-adic Heights on Curves

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Dedicated to Professor Kenkichi Iwasawa on the occasion of his 70th birthday

In this paper, we will present a new construction of the p-adic height pairings of Mazur-Tate [MT] and Schneider [S], when the Abelian variety in question is the Jacobian of a curve. Our aim is to describe the local height symbol solely in terms of the curve, using arithmetic intersection theory at the places not dividing p and integrals of normalized differentials of the third kind (Green's functions) at the places dividing p.

It is a pleasure to dedicate this note to Kenkichi Iwasawa, in thanks for the many inspiring things he has taught us.

§ 1. The local pairing

Let p be a rational prime and let $\boldsymbol{Q}_p$ denote the field of p-adic numbers. Let k be a non-archimedean local field of characteristic zero, with valuation ring $\mathcal{O}$, uniformizing parameter π, and residue field $\boldsymbol{F}=\mathcal{O}/\pi\mathcal{O}$ finite of order q. We fix a continuous homomorphism

$$\chi: k^* \longrightarrow \boldsymbol{Q}_p. \tag{1.1}$$

If the residue characteristic of k is not equal to p, then χ is trivial on the subgroup $\mathcal{O}^*$ and is determined by the value $\chi(\pi)$.

Let X be a complete non-singular, geometically connected curve defined over k, and assume for simplicity that X has a k-rational point. Let J denote the Jacobian of X over k. The following statement, as well as its proof, is similar to that of Proposition 2.3 in [G].

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