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Anderson-Ihara Theory: Gauss Sums and Circular Units

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Thrice the brinded cat hath mewed,
Thrice and once the hedge pig whined,
Harpier cries. 'tis time, 'tis time.

The Three Witches
Macbeth, Act IV Scene

Dedicated to Iwasawa on the occasion of his seventieth birthday

A few years ago, Ihara, [I], discovered a new sort of power series connected with the action of G_Q on the Tate-modules of Fermat curves of l-power degree. Since then Anderson, [A], refined and generalized these power series, interpreting them as analogues of the classical beta function. Moreover, once this analogy was made he naturally was forced to factor them into a product of three "gamma functions."

The previous paragraph is purposely vague and oversimplified. In this article I will attempt to make some of it a little less vague and indicate how the theory of these "gamma" and "beta" functions may be connected with and applied to other aspects of cyclotomy.

I. Ihara's "Beta" series

Let X_n denote the projective plane curve over Q determined by the homogeneous equation:

$$X^{l^n}+Y^{l^n}+Z^{l^n}=0.$$

Let J_n denote the Jacobian of X_n. We have natural maps, $X_{n+1}\to X_n$ and corresponding maps on the Jacobians. Hence we may define the G_Q-module

$$T=\varprojlim T_l(J_n(\bar{Q})).$$

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