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Explicit Universal Deformations of Galois Representations

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To K. Iwasawa on the occasion of his 70th birthday

Given a continuous absolutely irreducible representation

$$\bar{\rho}: G_{\boldsymbol{Q}} \longrightarrow GL_2(\boldsymbol{F}_p)$$

and a finite set of primes S which contains the primes of ramification for $\bar{\rho}$ and the prime number p, the notion of *universal deformation* for $(\bar{\rho}, S)$ was discussed in [M]. It was shown in [M] that there exists a complete noetherian local ring R with residue field $\boldsymbol{F}_p$, uniquely determined up to canonical isomorphism, and a lifting

$$\rho: G_{\boldsymbol{Q}} \longrightarrow GL_2(R)$$

of $\bar{\rho}$ (unique up to strict equivalence—see § 3.1 below) which is unramified outside S, and satisfies a universal property vis à vis all liftings of $\bar{\rho}$ to $GL_2(\mathscr{A})$ which are unramified outside S, where $\mathscr{A}$ ranges through the category of complete local noetherian rings with residue field $\boldsymbol{F}_p$.

For $S=\{p\}$ and a class of representations $\bar{\rho}$ ("special dihedral representations") the universal deformation ring R was shown to be a power series ring in 3 variables over $\boldsymbol{Z}_p$. If X is the "universal deformation space", i.e., the space of continuous homomorphisms from R to $\boldsymbol{Z}_p$, then X is a 3-dimensional analytic manifold over $\boldsymbol{Q}_p$ and for each $x \in X$ specialization of ρ yields a Galois representation

$$\rho_x: G_{\boldsymbol{Q}} \longrightarrow GL_2(\boldsymbol{Z}_p)$$

(determined up to strict equivalence) which is a lifting of $\bar{\rho}$ and is unramified outside S. One of the aims of [M] was to embark on a systematic study of certain "natural subspaces" in X: loci of points $x \in X$ such that

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