

## Vertex Operators in Conformal Field Theory on $\mathbb{P}^1$ and Monodromy Representations of Braid Group

*Dedicated to Professor Hiroshi Toda on his 60th birthday*

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### §0. Introduction

The 2-dimensional conformal field theory was initiated by A.A. Belavin, A.N. Polyakov and A.B. Zamolodchikov [BPZ] and was developed by many physicists, e.g. [DF], [ZF] etc. In the paper [BPZ], the significance of the primary fields for this theory is pointed out. V.G. Knizhnik and A.B. Zamolodchikov [KZ] developed the theory with current algebra symmetry, proposed the notion of primary fields with gauge symmetry, and gave the differential equations of multipoint correlation functions.

Our aim in this paper is to give rigorous foundations to the work of [KZ], and to reformulate and develop the operator formalism in the conformal field theory on the complex projective line  $\mathbb{P}^1$ . The space  $\mathcal{H}$  of operands is taken to be a sum  $\mathcal{H} = \sum_{j=0}^{\ell^2} \mathcal{H}_j$  of the integrable highest weight modules  $\mathcal{H}_j$  of the affine Lie algebra  $\hat{\mathfrak{g}} = \mathfrak{sl}(2, \mathbb{C}) \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c$  of type  $A_1^{(1)}$ . We fix the value  $\ell$  (positive integer) of the central element  $c$  of  $\hat{\mathfrak{g}}$  on the space  $\mathcal{H}$ . The Virasoro algebra  $\mathcal{L}$  acts on each  $\mathcal{H}_j$  through