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Hecke Algebra Representations of Braid Groups and Classical Yang-Baxter Equations

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Dedicated to Professor Itiro Tamura on his 60th birthday

Introduction

Let $\mathfrak{g}$ be a simple Lie algebra over $\boldsymbol{C}$ and let c be the Casimir element. Motivated by the study of rational solutions of the classical Yang-Baxter equations due to Belavin and Drinfel'd [BD], we shall construct a flat connection over

$$X_n=\{(z_1, \cdots, z_n) \in \boldsymbol{C}^n;\ z_i \neq z_j \ \text{ if } i\neq j\}.$$

Let ρ_i: $\mathfrak{g}\to\operatorname{End}(V_i)$, $1\leq i\leq n$, be irreducible representations. Putting $\Omega=2^{-1}(\Delta c-c\otimes 1-1\otimes c) \in U(\mathfrak{g})\otimes U(\mathfrak{g})$, we define $\Omega_{ij} \in \operatorname{End}(V_1\otimes\cdots\otimes V_n)$ by $\Omega_{ij}=(\rho_i\otimes\rho_j)(\Omega)$. For a complex number λ we consider the connection ω defined by

$$\sum_{1\leq i<j\leq n} \lambda\Omega_{ij} d\log(z_i-z_j).$$

The integrability of this connection follows from the following relations, which we shall call the infinitesimal pure braid relations.

$$\begin{aligned} &[\Omega_{ij}, \Omega_{ik}+\Omega_{jk}]=[\Omega_{ij}+\Omega_{ik}, \Omega_{jk}]=0 \qquad \text{for } i<j<k, \\ &[\Omega_{ij}, \Omega_{kl}]=0 \qquad \text{for distinct } i, j, k, l. \end{aligned} \tag{0.1}$$

Thus as the monodromy of our connection we obtain a linear representation of the fundamental group of X_n, which is the pure braid group on n strings. If all the representation ρ_i are the same, the above construction gives a linear representation of the braid group depending on a parameter λ.

In the preceding paper [K3], we have shown that in the case $\mathfrak{g}=\mathfrak{sl}(2, \boldsymbol{C})$ and its two dimensional irreducible representation, the linear representation of the braid group obtained in the above manner is the so called Pimsner-Popa-Temperley-Lieb representation appearing in works of Jones [J]. The main theme of this note is to generalize this result to