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Lectures on Conformal Field Theory

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Lecture 1.

In statistical physics, in the theory of critical phenomena at the second order phase transition points, the global scaling symmetry has been known and extensively used for many years. This led to the renormalization group approach of Wilson to the critical phenomena.

In the scaling theory, which is the theory at the second order phase transition point, a given physical system is classified by a set of basic operators $\{\phi_i(x)\}$, having anomalous dimensions $\{\Delta_i\}$. This is the spectrum of the theory. Under scaling transformations of the space

$$x \longrightarrow \lambda x \tag{1.1}$$

(λ being a constant parameter) the basic operators transform like:

$$\phi_i(x) \longrightarrow \lambda^{\Delta_i} \phi_i(\lambda x). \tag{1.2}$$

It means that if we made the transformation (1.2) then the correlation functions for the basic operators

$$\langle \phi_1(x_1) \phi_2(x_2) \cdots \rangle \tag{1.3}$$

will not change. In particular, this fixes uniquely the 2-point function

$$\langle \phi_i(x_1) \phi_i(x_2) \rangle = \frac{\text{const}}{|x_1 - x_2|^{2\Delta_i}} \tag{1.4}$$

(the normalizing constant is arbitrary), so, the anomalous dimensions Δ_i can be found, if the two-point functions are known. By anomalous dimension we mean that it is different from canonical dimensions for free, noninteracting fields. The general situation for the critical phenomena is that the basic fields (operators) will be strongly interacting. And it is reflected in the fact that their dimensions, defined by two-point functions like (1.4), will be anomalous.

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