Advanced Studies in Pure Mathematics 16, 1988
Conformal Field Theory and Solvable Lattice Models
pp. 17–122

Exactly Solvable SOS Models II:

Proof of the star-triangle relation and combinatorial identities

E. Date, M. Jimbo, A. Kuniba, T. Miwa and M. Okado

Dedicated to Professor Nagayoshi Iwahori on his 60th birthday

§ 1 Introduction

This is a continuation of the paper [1], hereafter referred to as Part I. As announced therein, we give detailed proofs of (i) the star-triangle relation (STR) for the restricted solid-on-solid (SOS) models [2], and (ii) the combinatorial identities used in the evaluation of the local height probabilities (LHPs) [1, 3]. We try to keep this paper self-contained so that the mathematical content is comprehensible without reading Part I. Below we shall outline the setting and the content of each section.

1.1. The fusion models

The STR is the following system of equations for functions $W(a, b, c, d|u)$ ($a, b, c, d \in \boldsymbol{Z}$, $u \in \boldsymbol{C}$), to be called Boltzmann weights:

$$\sum_g W(a, b, g, f|u)\ W(f, g, d, e|u+v)\ W(g, b, c, d|v)$$
$$=\sum_g W(f, a, g, e|v)\ W(a, b, c, g|u+v)\ W(g, c, d, e|u).$$

Section 2 deals with the construction of solutions to the STR by the fusion procedure. Using the Boltzmann weights of the eight vertex SOS (8VSOS) model [4] as an elementary block, we construct "composed blocks" satisfying the STR. As in the 8VSOS case the resulting weights depend on the elliptic "nome" p as well as the parameter u.

This construction given in section 2.1 is known as the block spin transformation in the renormalization group theory. Namely, we sum up the freedoms $\ell_i \in \boldsymbol{Z}$ associated with sites i of a given lattice $\mathscr{L}_1$, leaving free the ones in $\mathscr{L}_N = N\mathscr{L}_1$. (See Fig. 1.1.) The ℓ_i is called a height in the sequel. In general, the locality of the Hamiltonian is not preserved by this

Received March 27, 1987.
Revised June 2, 1987.