Advanced Studies in Pure Mathematics 16, 1988
Conformal Field Theory and Solvable Lattice Models
pp. 1–15

Scaling Limit Formula for 2-Point Correlation Function of Random Matrices

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In this article we give some results about 1 and 2 point correlation functions of the Gibbs measure of random matrices

(0.1) $$\Phi d\tau = \Phi dx_1 \wedge \cdots \wedge dx_N$$

with the weight function $\Phi = \exp(-1/2(x_1^2 + \cdots + x_N^2)) \prod_{1 \leq j < k \leq N} |x_j - x_k|^\lambda$ for a constant $\lambda > 0$. As in [A1] we use the notations $(j, k) = x_j - x_k$, $d\tau_{N,p} = dx_{p+1} \wedge \cdots \wedge dx_N$ (which means a differential $(N-p)$-form) for $0 \leq p < N$. We put $n = N - p$. We consider more generally the density

(0.2) $$\Phi_{N,p} = \exp\left(-\frac{1}{2}(x_1^2 + \cdots + x_N^2)\right) \sum_{1 \leq \mu < \nu \leq n} |x_{p+\mu} - x_{p+\nu}|^\lambda \cdot \prod_{j=1}^{p} \prod_{1 \leq \mu \leq n} |x_{p+\mu} - x_j|^{\lambda'_j}$$

on the Euclidean space $\boldsymbol{R}^{N-p}$ of the variables $x_{p+1}, \cdots, x_N$. Here $\lambda'_1, \cdots, \lambda'_p$ denote some positive constants. For $\varepsilon_j = \pm 1$ we denote by $\langle (i_1, j_1)^{\varepsilon_1} \cdots (i_l, j_l)^{\varepsilon_l} | \lambda'_1, \cdots, \lambda'_p \rangle$ the correlation functions

(0.3) $$\int_{\boldsymbol{R}^n} (i_1, j_1)^{\varepsilon_1} \cdots (i_l, j_l)^{\varepsilon_l} \Phi_{N,p} d\tau_{N,p}.$$

We abbreviate it by $\langle (i_1, j_1)^{\varepsilon_1} \cdots (i_l, j_l)^{\varepsilon_l} \rangle$ if $\lambda'_1 = \cdots = \lambda'_p = 0$. This is a l-point correlation function for the density $\Phi d\tau$.

The reduced density of p points

(0.4) $$F_{N,p} = \int_{\boldsymbol{R}^n} \Phi_{N,p} d\tau_{N,p}$$

is known to be analytic in $x_1, \cdots, x_p$ and $\lambda, \lambda'_1, \cdots, \lambda'_p$. However the following problem seems difficult and interesting:

Received December 18, 1986.