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Multi-Tensors of Differential Forms on the Hilbert Modular Variety and on Its Subvarieties, II

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Dedicated to Prof. Ichiro Satake and Prof. Friedrich Hirzebruch on their sixtieth birthdays

Let Γ_K denote the Hilbert modular group associated with a totally real algebraic number field K of degree $n>1$. Let X_K be the Hilbert modular variety H^n/Γ_K. The present paper is the continuation of a study [8], and our purpose is to extend the known range of K for which an assertion (☆) holds where

(☆) *any subvariety in X_K of codimension one is of general type.*

We show that *if $n\geq 3$, then (☆) holds only with finite exceptions.* It was shown in our previous paper [8] that if the dimension $n\geqq 3$ is fixed, then (☆) holds with finite exceptions. The main theorem of the present paper is as follows:

Theorem. (☆) *holds if $n>26$, or if $n>14$ and the ideal in the maximal order of K generated by 2 is unramified at any prime of degree one.*

As stated in [8], (☆) has the consequent on the property of X_K which we restate here for reader's convenience.

(I) *Let X_K° denote the smooth locus of X_K, and let $\tilde{X}_K$[1)] be any smooth variety having X_K° as an open subset. Then for any birational morphism φ of $\tilde{X}_K$ to a smooth variety, $\varphi|_{X_K^\circ}$ gives rise to an open embedding.*

(II) *The birational automorphism group of X_K (or equivalently, the automorphism group of the Hilbert modular function field over $\boldsymbol{C}$) is equal to the automorphism group of X_K, which is canonically isomorphic to a semidirect product $H_K^{(2)}\rtimes \mathrm{Aut}\,(K/\boldsymbol{Q})$ where $H_K^{(2)}=\{x\in H_K \mid x^2=1\}$, H_K denoting the ideal class group of K in the narrow sense.*

As we see in §2, in order to prove Theorem we need to show

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1) ~ is missing in [8], Cor. 1, p. 660.