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Invariants and Hodge Cycles

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Dedications

I would like to dedicate this paper to Professor I. Satake and Professor F. Hirzebruch on their birthdays, whose mathematics greatly influenced me.

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§ 1. Introduction

Let F be an even $2N$ dimensional vector space over $\boldsymbol{Q}$, and β be a non degenerate alternating bilinear form on F. We put

$$F_{\boldsymbol{R}}=F\otimes\boldsymbol{R}\cong\boldsymbol{R}^{2N},$$
$$Sp(F,\beta)=\{g\in\operatorname{aut}(F)\,|\,\beta(gu,gv)=\beta(u,v)\},$$

and

$$\mathfrak{H}(F,\beta)=\left\{J\in\operatorname{aut}(F_{\boldsymbol{R}})\left|\begin{array}{l}J^2=-1\\ \beta(u,Jv)=\text{symmetric on } u,v\\ \beta(u,Ju)>0 \text{ for } 0\neq{}^{\forall}u\in F\end{array}\right.\right\};$$

and we call $\mathfrak{H}(F,\beta)$ the Siegel half space.

An element J of $\mathfrak{H}(F,\beta)$ is a complex structure of the real vector space $F_{\boldsymbol{R}}$, therefore it defines a complex vector space $(F_{\boldsymbol{R}},J)$ of N dimension, which we denote by E or E_J. The group $Sp(F,\beta)$ operates on $\mathfrak{H}(F,\beta)$ by

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