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T-Complexes and Ogata's Zeta Zero Values

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Dedicated to Prof. Ichiro Satake and Prof. Friedrich Hirzebruch on their sixtieth birthdays

Introduction

In [T1], Tsuchihashi defined the notion of cusp singularities in arbitrary dimension. They include the Hilbert modular cusp singularities as a special case. In this paper, we will show the rationality of the zeta zero value $Z(C, \Gamma; 0)$ of the zeta function associated to a Tsuchihashi cusp singularity which was defined by Ogata [Og]. He gave a formula for the zero value as a sum of integrals of C^∞-functions described by the characteristic function of the convex cone C. By this formula, he showed that the value is a half-integer in odd-dimensional case [Og, Theorem 2.3]. In two dimensional case, the singularity is a Hilbert modular cusp and 12 times the zeta zero value is an integer by [Z].

By the construction of Tsuchihashi cusp singularities, they have toroidal resolutions and the exceptional sets are toric divisors in the sense of [S2]. In order to describe toric divisors, we introduce the notion of *T-complexes* which is essentially equal to that of the weighted dual graphs which appear in [T1]. A T-complex Σ is a category with a finite number of objects. We define a functor D^0_Q from Σ to the category of $\boldsymbol{Q}$-vector spaces. We show that the rational number field $\boldsymbol{Q}$ has a natural injection into the inductive limit $\operatorname{ind\,lim}_\Sigma D^0_Q$. We define a special element ω_Σ of $\operatorname{ind\,lim}_\Sigma D^0_Q$. When Σ is the T-complex associated to a toroidal resolution of a Tsuchihashi cusp singularity (C, Γ), Ogata's formula means that there exists an explicit retraction $\operatorname{ind\,lim}_\Sigma D^0_Q \otimes \boldsymbol{R} \to \boldsymbol{R}$ and the zero value $Z(C, \Gamma; 0)$ is the image of ω_Σ in $\boldsymbol{R}$. By using an equality in Section 1 for a nonsingular complete fan, we show that ω_Σ is in the image of $\boldsymbol{Q}$ in $\operatorname{ind\,lim}_\Sigma D^0_Q$ for any T-complex Σ. This implies that the image of ω_Σ in $\boldsymbol{R}$ is independent of the retraction and is a rational number.

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