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A Formula for the Dimension of Spaces of Cusp Forms of Weight 1

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Dedicated to Prof. Ichiro Satake on his sixtieth birthday

Introduction

Let Γ be a fuchsian group of the first kind and denote by d_1 the space of cusp forms of weight 1 on the group Γ. It would be interesting to have a certain formula for d_1. But it is not effective to compute the dimension d_1 by means of the Riemann-Roch theorem. The purpose of this paper is to give some formula of d_1 by making use of the Selberg trace formula ([4], [6], [7]).

§ 1. The Selberg eigenspace

Let S denote the complex upper half-plane and we put $G=SL(2, \boldsymbol{R})$. Consider direct products

$$\tilde{S}=S\times T,\ \tilde{G}=G\times T,$$

where T denotes the real torus. The operation of an element (g, α) of $\tilde{G}$ on $\tilde{S}$ is represented as follows:

$$\tilde{S}\ni (z, \phi)\longrightarrow(g, \alpha)(z, \phi)=\left(\frac{az+b}{cz+d}, \phi+\arg(cz+d)-\alpha\right)\in \tilde{S},$$

where $g=\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in G$. The space $\tilde{S}$ is a weakly symmetric Riemannian space with the $\tilde{G}$-invariant metric

$$ds^2=\frac{dx^2+dy^2}{y^2}+\left(d\phi-\frac{dx}{2y}\right)^2,$$

and with the isometry μ defined by $\mu(z, \phi)=(-\bar{z}, -\phi)$. The $\tilde{G}$-invariant measure $d(z, \phi)$ associated to the $\tilde{G}$-invariant metric is given by

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