Advanced Studies in Pure Mathematics 15, 1989
Automorphic Forms and Geometry of Arithmetic Varieties
pp. 281–286

Any Irreducible Smooth GL_2-Module is Multiplicity Free for any Anisotropic Torus

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Dedicated to Prof. Ichiro Satake on his sixtieth birthday

§ 1.

Let k be a non-archimedean local field, B be a quaternion algebra, i.e. a central simple algebra of rank 4 over k. Let L be a separable quadratic subfield of B. The group $G=B^\times$, of the regular elements of B, is a T.D.L.C. (= totally disconnected locally compact) group by the induced topology from B, and $H=L^\times$ is a closed subgroup of G. In other words, G is a k-form of GL_2, and H is a maximal torus anisotropic modulo center. Let (π, E) be a smooth representation of G on the complex vector space E. The purpose of this paper is to prove the following:

Theorem A. *If (π, E) is irreducible as G-module, then it is multiplicity free as H-module. Namely, there is a subset $\hat{H}(\pi)$ of the set $\hat{H}$ of all quasicharacters of H such that*

$$\pi = \bigoplus_{\chi \in \hat{H}(\pi)} \chi \qquad \textit{as H-module.}$$

§ 2.

The irreducible smooth representations of $G=B^\times$ are classified into several series (cf. [J-L], [K] for split G, and [G-G], [Ho] for non-split G). To identify the set $\hat{H}(\pi)$ for all L amounts to get a complete knowledge for the representation π, at least character-theoretically. In this respect, there are no difficulties if k has odd residual characteristic. While, in dyadic case, I have determined $\hat{H}(\pi)$ (for all L) for some series of π's, but not yet for all series.

When G is non-split, i.e. B is a division algebra, there is a close connection between Theorem A and the Basis Problem of modular forms as

Received March 27, 1987.