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Zeta Functions of Finite Graphs and Representations of p-Adic Groups

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Dedicated to Prof. Friedrich Hirzebruch and Prof. Ichiro Satake on their sixtieth birthdays

Contents

§ 0. Introduction

0-1. In this paper we shall be concerned with the two different subjects, which have been developed separately. One is a combinatorial problem in algebraic graph theory, and the other is an arithmetic of discrete subgroups of p-adic groups and their representations.

Suppose that X is a finite (multi) graph, which is not a tree. We always assume that X is *non-oriented*. A closed path C in X is called reduced, if C and $C^2 = C.C$ have no backtracking. Then obviously the set $\mathscr{C}_l^{red}(X)$ of reduced closed paths of length l is finite, and $\#(\mathscr{C}_l^{red}(X)) \to \infty$ $(l \to \infty)$ if X is not homotopic to a circuit, i.e., S^1. (See § 1 for

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