Advanced Studies in Pure Mathematics 15, 1989
Automorphic Forms and Geometry of Arithmetic Varieties
pp. 171–210

Selberg-Ihara's Zeta function for p-adic Discrete Groups

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Dedicated to Prof. Friedrich Hirzebruch and Prof. Ichiro Satake on their sixtieth birthdays

Contents

Introduction

0-1. Let G be $\mathrm{PSL}(2, \boldsymbol{R})$ and let Γ $(\subset G)$ be a Fuchsian group of the first kind. In [Sel], a zetafunction $Z_\Gamma(s)$ was introduced and proved to have many important properties which resemble those of usual L-functions, such as Euler product, functional equation, and analogue of Riemann Hypothesis. This function, now called with the name of Selberg, is generalized to any discrete subgroup Γ of a semi-simple Lie group of $\boldsymbol{R}$-rank one, when G/Γ is compact by Gangolli [Gan], and later by Gangolli-Warner [G-W] to the case when G/Γ has a finite volume. Meanwhile, an analogue of $Z_\Gamma(s)$ was introduced by Ihara [I-1], for a cocompact torsion-free discrete subgroup Γ of $\mathrm{PSL}(2, K)$ or $\mathrm{PL}(2, K)$, where K is a p-adic field. Especially it was shown that Ihara's zeta function $Z_\Gamma(u)$ is a

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Received November 30, 1987.
Revised April 1, 1988.
*) The first author has been partially supported by Sonderforschungsbereich 170, 'Geometrie und Analysis', at Univ. Göttingen.