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Cusps on Hilbert Modular Varieties and Values of L-Functions

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§1.

Let s be a cusp, and $D=\sum S_\tau$ the corresponding cusp divisor on a Hilbert modular variety X. Every such a cusp belongs to a pair (M, V) where M is a lattice (isomorphic to $\boldsymbol{Z}^n$), and V a group of units (isomorphic to $\boldsymbol{Z}^{n-1}$) in a totally real number field F of degree n over $\boldsymbol{Q}$, subject to the restriction that all elements in V are totally positive, and that V acts on M by multiplication, $VM=M$. However, the cusp divisor D is not unique for a given pair (M, V).

The divisor D is a normal crossing divisor, i.e. the irreducible components S_τ (hypersurfaces on X) intersect only in simple normal crossings. The complicated intersection behavior of the S_τ can be described in terms of a triangulation of the $(n-1)$-torus $\boldsymbol{R}^{n-1}/V$. Every hypersurface S_τ corresponds to a vertex τ of this triangulation, and k different hypersurfaces S_{τ_j} $(1\le j\le k)$ intersect either in a $(n-k)$-dimensional submanifold S_σ, or the intersection set is empty. In the first case, σ is the unique simplex of the triangulation having the τ_j as vertices.

This description of the cusp divisor D was given for the first time by Hirzebruch [4] in the case of a real quadratic field F $(n=2)$. He showed in particular that the corresponding triangulation of the torus $S^1=\boldsymbol{R}/V$ is given by the continued fraction expansion of a quadratic irrationality associated with M. In the same paper, Hirzebruch defined a rational number $\varphi(s)=\varphi(M, V)$ called the signature defect of s, in the following way: let Y be a small closed neighbourhood in X of the cusp s. Then Y is a manifold with boundary ∂Y which is a $T^n=\boldsymbol{R}^n/M$ bundle over the torus $T^{n-1}=\boldsymbol{R}^{n-1}/V$ completely determined by the pair (M, V). Let $L(Y)$ be the L-polynomial in the relative Chern classes of Y, and $\operatorname{sign}(Y)$ the signature of Y. From the signature theorem [1], it follows that

$$\varphi(M, V) := L(Y)-\operatorname{sign}(Y)$$

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