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Zeta Functions Associated to Cones and their Special Values

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Introduction

The purpose of this paper is to give a survey on zeta functions associated to (self-dual homogeneous) cones and their special values, including some recent results of ours on this subject.

In § 1 we summarize basic facts on self-dual homogeneous cones and the associated Γ-functions. § 2 is concerned with the zeta functions. Let V be a real vector space, $\mathscr{C}$ a self-dual homogeneous cone in V, and let G be the automorphism group $\mathrm{Aut}(V, \mathscr{C})^{\circ}$. We fix a $\boldsymbol{Q}$-simple $\boldsymbol{Q}$-structure on $(V, \mathscr{C})$. As is well-known, the pair (G, V) is a "prehomogeneous vector space" in the sense of Sato-Shintani [SS]. Following the general idea in [SS], we define a set of zeta functions $\{\xi_I\}$, each one of which is associated to a connected component V_I of $V^{\times}=V-S$, S denoting the singular set; in particular, $\xi_{(0)}=Z_{\mathscr{C}}$ is the zeta function associated to the cone $V_{(0)}=\mathscr{C}$. Then we give an explicit expression for the system of functional equations (Theorems 2.2.2, 2.3.3). Under the assumption that *d is even*, taking suitable linear combinations of these zeta functions, we define a new kind of L-functions L_I, which are shown to satisfy individually (or two in a pair, according to the cases) a functional equation of ordinary type (see (2.3.5)). We give some (new) results (Theorems 2.3.9, 2.4.1) on the residues and special values of these zeta and L-functions, where two extreme L-functions $L_{(0)}$ and $L_{(r_1)}$ play an essential role. These extreme L-functions, which generalize the (partial) Dedekind zeta function and the Shimizu L-function in the Hilbert modular case, seem to be of particular importance from the number-theoretic view point.

In § 3, we consider the corresponding (rational) symmetric tube domain $\mathscr{D}=V+\sqrt{-1}\mathscr{C}$ and, under an additional assumption that the ***$\boldsymbol{Q}$-rank of G is one***, study the geometric invariants (χ_{∞}, τ_{∞}, etc.) associated to the cusp singularities appearing in the (standard) compactification of the arithmetic quotient space $\tilde{\Gamma}\backslash\mathscr{D}$ ([S3, 4]). A typical example is the Hilbert

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