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A Realization of Semisimple Symmetric Spaces and Construction of Boundary Value Maps

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§ 0. Introduction

A homogeneous space G/H is called a semisimple symmetric space if G is a real connected semisimple Lie group and there exists an involution of G such that H is an open subgroup of the fixed point group of the involution. The most fundamental problem on the harmonic analysis on G/H is to give an explicit decomposition of $L^2(G/H)$ into irreducible representations of G, that is, to get a Plancherel formula for $L^2(G/H)$. Here $L^2(G/H)$ is the space of square integrable functions on G/H with respect to the invariant measure. In [O3] I proposed a method to obtain the Plancherel formula. The method explained there works well for the most continuous spectra on $L^2(G/H)$ with respect to the ring $D(G/H)$ of invariant differential operators on G/H, and the Plancherel measure for the spectra is expressed by "c-function" for G/H which is explicitly calculated by the method in [O3, §8], where G^d/H^d should be corrected to G^d/K^d. Comparing to the continuous spectra, the discrete spectra (the discrete series for G/H) are not easy to analyse by the method mentioned in [O3, § 9] and it is hard to get the precise parametrization of the discrete series or to investigate its structure especially in the case when the symmetric space is not a K_ε-type. On the other hand, by using Flensted-Jensen's duality method, we can directly study the discrete series and in fact we get sufficient informations to analyse the discrete series ([F] and [MO] etc.). Applying the usual method of parabolic induction for repre-

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