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Asymptotic Behavior of Spherical Functions on Semisimple Symmetric Spaces

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Contents

§ 0. Introduction

Let G be a connected real semisimple Lie group, σ an involution of G, and H an open subgroup of the fixed point group G^σ. Then the homogeneous space G/H is called a semisimple symmetric space. In this paper, a K-finite simultaneous eigenfunction of the invariant differential operators on G/H is called a spherical function, where K is a maximal compact subgroup of G modulo center. It is known that such a spherical function has an asymptotic expansion at infinity, which really converges, as is shown by [HC] and [CM] in the group case and by [Ba] and [O3] in general cases. In this paper, we will give the main non-vanishing terms in the expansion, that is, the growth order at infinity, by using some geometric interpretation. It plays an important role for the harmonic analysis on G/H.

The idea here is similar as in [MO], where we describe discrete series for G/H. But we get a better result here than [MO, Lemma 1] which is essential in [MO] and we can simplify the proof of the main theorem in [MO]. In fact we can omit complicated arguments according to the classification of root systems. The simpler proof is given in [Ma2]. Moreover for a given representation of G realized on a function space on G/H, we can tell in which principal series for G/H the representation is imbedded.

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