Advanced Studies in Pure Mathematics 14, 1988
Representations of Lie Groups, Kyoto, Hiroshima, 1986
pp. 531–540

A Description of Discrete Series for Semisimple Symmetric Spaces II

Toshihiko Matsuki

§ 1. Introduction

In [F], Flensted-Jensen constructed countably many discrete series for a semisimple symmetric space G/H when

$$\text{rank}(G/H)=\text{rank}(K/K\cap H). \tag{1.1}$$

Conversely, [OM1] proved that (1.1) holds if there exist discrete series for G/H. Moreover [OM1] constructed Harish-Chandra modules B_λ^j which parametrize all the discrete series for G/H, where j runs through finite indices and λ runs through lattice points contained in a positive Weyl chamber. In this paper, we give a necessary condition for j and λ so that the module B_λ^j is nontrivial. In the subsequent paper [OM2], we will prove that the condition also assures $B_\lambda^j\neq\{0\}$. We remark that our results also covers "limits of discrete series" for G/H. In the appendix, we give a certain simplification of the proof of a main result in [OM1]. To state the precise result in this paper, we prepare some notations.

Let $\mathfrak{g}$ be a semisimple Lie algebra and σ an involution (automorphism of order 2) of $\mathfrak{g}$. Fix a Cartan involution θ of $\mathfrak{g}$ such that $\sigma\theta=\theta\sigma$. Let $\mathfrak{g}=\mathfrak{h}+\mathfrak{q}$ (resp. $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$) be the decomposition of $\mathfrak{g}$ into the $+1$ and -1 eigenspaces for σ (resp. θ). Let $\mathfrak{g}_c$ denote the complexification of $\mathfrak{g}$ and put

$$\mathfrak{k}^d=\mathfrak{k}\cap\mathfrak{h}+\sqrt{-1}(\mathfrak{p}\cap\mathfrak{h}),\qquad \mathfrak{p}^d=\sqrt{-1}(\mathfrak{k}\cap\mathfrak{q})+\mathfrak{p}\cap\mathfrak{q},$$
$$\mathfrak{h}^d=\mathfrak{k}\cap\mathfrak{h}+\sqrt{-1}(\mathfrak{k}\cap\mathfrak{q}),\qquad \mathfrak{q}^d=\sqrt{-1}(\mathfrak{p}\cap\mathfrak{h})+\mathfrak{p}\cap\mathfrak{q},$$
$$\mathfrak{g}^d=\mathfrak{k}^d+\mathfrak{p}^d=\mathfrak{h}^d+\mathfrak{q}^d.$$

Let G_c be a connected complex Lie group with Lie algebra $\mathfrak{g}_c$, and let G, K, H, G^d, K^d, H^d, H_c and K_c be the analytic subgroups of G_c corresponding to $\mathfrak{g}$, $\mathfrak{k}$, $\mathfrak{h}$, $\mathfrak{g}^d$, $\mathfrak{k}^d$, $\mathfrak{h}^d$, $\mathfrak{h}_c$ and $\mathfrak{k}_c$, respectively.

In [OM1], we studied the discrete series for G/H and proved that

Received April 6, 1987.