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Cohomological Hardy Space for $SU(2,2)$

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Introduction

Let G be a connected real semisimple linear Lie group and let P be a parabolic subgroup. Let G_C and P_C be the complexification of G and P respectively. Our aim is to find a good description of relations between the G-orbits of G_C/P_C and subquotients of degenerate principal series. In this article we treat an example for the group $SU(2, 2)$.

Let $G=SU(2, 2)$ and $K=S(U(2)\times U(2))$. Let P be a parabolic subgroup of G such that G/P is Shilov boundary of G/K. Then G/P is a unique closed G-orbit of G_C/P_C and there exist three open G-orbits of G_C/P_C. Two open orbits are isomorphic to G/K as G-homogeneous space. But in this article we consider the other orbit. This orbit is isomorphic to a semisimple symmetric space $SU(2,2)/S(U(1, 1)\times U(1, 1))$. We call this orbit $\bar{D}$. We consider the homogeneous line bundle L corresponding to the representation in unitary degenerate series with "the most singular parameter". We can get a holomorphic homogeneous line bundle on G_C/P_C whose restriction to G/P is L. We denote this line bundle and the sheaf of its holomorphic sections by the same letter L. We investigate some relation between the Čech cohomology group $H^2(\bar{D}, L)$ and a decomposition of the above degenerate series representation in Kashiwara and Vergne [KV]. Although the K-type of this cohomology group is known by the very general result of Rawnsley, Schmid, and Wolf [RSW], our approach is purely geometric and we construct an injective G-equivariant "boundary map" of the cohomology space to the space of hyperfunction-section of L on G/P using a Mayer-Vietris exact sequence. We remark this construction of the boundary map is applicable in the case of $SO_0(n, 2)$.

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