Advanced Studies in Pure Mathematics 14, 1988
Representations of Lie Groups, Kyoto, Hiroshima, 1986
pp. 417–468

Algebraic Structures on Virtual Characters of a Semisimple Lie Group

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Introduction

Let G be a connected semisimple Lie group with finite centre. Take a continuous representation π of G on a Hilbert space $\mathfrak{H}$. For a maximal compact subgroup K of G, put

$$\mathfrak{H}_K=\{v \in \mathfrak{H} \mid \dim \pi(K)v<\infty\}.$$

We call a vector v in $\mathfrak{H}_K$ a K-finite vector. A representation $(\pi, \mathfrak{H})$ is called admissible if, for any irreducible representation δ of K, the multiplicity of δ in $\mathfrak{H}_K$ is finite. All the irreducible admissible representations of G are classified under infinitesimal equivalence. There are at least three different methods to classify them. Namely, Langlands' classification ([30]), classification using the theory of D-modules ([2]) and Vogan's minimal K-type arguments ([41]). However, we still cannot understand the admissible representations well. For example, Langlands' parameter of a finite dimensional representation is very difficult to calculate out. So we need not only the classification theory but also more easier description or structures of admissible representations. There are many improvements in this direction, for example, theory of primitive ideals, K_C-orbits on flag varieties, Weyl group representations on virtual character modules and so on.

We treat here algebraic structures on virtual character modules. In [28, Appendix], G. J. Zuckerman defined a representation of Weyl groups on virtual character modules on G with regular infinitesimal character. This Weyl group representation provides powerful methods to calculate invariants of admissible representations, such as Gelfand-Kirillov dimensions, τ-invariants, primitive ideals and so on, and to classify them in large. In part I of this paper, we improve his definitions and define a representation of a Hecke algebra on a virtual character module with singular infinitesimal character. After this, we show how useful the representations of Weyl groups and their Hecke algebras are in invariant theory of repre-

Received February 9, 1987.