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Irreducible Unitary Representations of the Group of Maps with Values in a Free Product Group

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Dedicated to Professor R. Takahashi for his 60th birthday

Introduction

In [30], Vershik, Gelfand and Graev studied the construction of irreducible unitary representations of the group $C^\infty(X, G)$ of smooth maps of a compact manifold X with values in a Lie group G. Following the physical terminology such a group is called a current group. In case $G=SL(2, \boldsymbol{R})$, they afforded factorizable irreducible unitary representations of the current group which depend upon measures on X. Their method reveals that the structure of a measure space is important rather than the structure of a manifold. In fact they started with the construction of those representations of the weak current group $G^{(X)}$. A weak current group is the group of maps of a measurable space X with only finitely many values in a topological group G. Furthermore their method relies deeply on the structure of the neighborhood of the trivial representation of $G=SL(2, \boldsymbol{R})$. In other words, it is essential that there exists a canonical state on $SL(2, \boldsymbol{R})$ (see [32] for its definition).

Apart from the representation theory of current groups, there has been a remarkable progress in harmonic analysis on free groups. In [10], Figà-Talamanca and Picardello found a close resemblance between harmonic analysis on free groups and that of $SL(2, \boldsymbol{R})$. Their results are known to be extended to certain free product groups (cf. [15]).

Based on the above stated resemblance, we consider in this paper the construction of factorizable irreducible unitary representations of the weak current group $G^{(X)}$. Here X is a measurable space and G is the free product of a countable family $(G_i)_{i\in I}$ of countable groups. Note that if all G_i are infinite cyclic then G is a free group. In Section 1 we show that a length function ℓ on G is negative definite, which yields a canonical state $\psi_t(x)=t^{\ell(x)}$ where $x\in G$ and $0<t<1$. The cyclic unitary representation L_t defined by ψ_t is called the canonical representation. We remark

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