

Character, Character Cycle, Fixed Point Theorem and Group Representations

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§ 0. Introduction

Among many methods to derive Weyl's character formula, there is an application of the fixed point theorem (à la Atiyah Singer) to a line bundle on the flag variety. Namely, any finite-dimensional irreducible representation of a reductive group G is obtained as the cohomology group of an equivariant line bundle on the flag variety. Hence the trace of the action of an element g of G is obtained as the sum of the contributions at each fixed point. When g is a regular element, there are as many fixed points as the order of the Weyl group and each of them gives one of the terms $\text{sgn } w e^{w\lambda} / \prod (e^{\alpha/2} - e^{-\alpha/2})$ in Weyl's character formula.

On the other hand, Harish-Chandra [HC] defined the character of an (infinite-dimensional) representation of a real semisimple group $G_{\mathbb{R}}$ as an invariant eigendistribution. In this paper we shall give a character formula in terms of the geometry of flag manifold as a conjecture and prove it for discrete series. The correspondence of Harish-Chandra modules and K -equivariant sheaves is completed by adding representations of $G_{\mathbb{R}}$ and $G_{\mathbb{R}}$ -equivariant sheaves (See [K₂] and also the articles of W. Schmid and J. Wolf in the same volume). Then the character would be calculated from $G_{\mathbb{R}}$ -equivariant sheaves. We can illustrate this schematically as follows.

