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Some Remarks on Discrete Series Characters for Reductive *p*-adic Groups

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§ 0. Introduction

Let F be a p-adic field of characteristic zero (a finite extension of Q_p), and let G be the subgroup of F-rational points of a connected, reductive algebraic group defined over F. A complex representation (π, V) of G is *smooth* if, for each $v \in V$, there is an open subgroup K_v of G which fixes v. For any open subgroup K of G, define V^K to be the set of K fixed vectors in V. The representation (π, V) is *admissible* if (1) π is smooth; (2) dim V^K is finite for all compact open subgroups of G.

The basic theorem concerning admissible representations was proved by Bernstein.

Theorem 0.1 (Bernstein [B]). *If $(\pi, \mathscr{H})$ is an irreducible unitary representation of G, then π is admissible.*

This is to be interpreted as follows. Let V^∞ be the subspace of π-smooth vectors in the Hilbert space $\mathscr{H}$. Then V^∞ is dense in $\mathscr{H}$, and we say that $(\pi, \mathscr{H})$ is admissible if (π, V^∞) is admissible according to the above definition.

If (π, V) is an admissible representation of G, and $f \in C_c^\infty(G)$, the space of locally constant, compactly supported, complex-valued functions on G, then

(0.2)
$$\pi(f)=\int_G f(x)\pi(x)dx$$

is an operator of finite rank. (Here, dx is a Haar measure on G.) The map

(0.3)
$$f \longmapsto \hat{f}(\pi)=\text{trace } \pi(f)$$

is called the *distribution character* of π.

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