Advanced Studies in Pure Mathematics 14, 1988
Representations of Lie Groups, Kyoto, Hiroshima, 1986
pp. 257–287

Schur Orthogonality Relations for Non Square Integrable Representations of Real Semisimple Linear Group and Its Application

Hisaichi Midorikawa

Introduction

In the previous paper [20], we discuss the Schur orthogonality relations for certain non square integrable representations of a given connected real semisimple linear group G. Those representations are the subrepresentations of unitary principal series of G induced from a maximal cuspidal parabolic subgroup, although I did not state explicitly this fact in [20]. We formulate our results as follows.

Let $C^\infty(G)$ be the set of all complex valued C^∞-functions on G and $\mathfrak{g}_C$ the complexification of the Lie algebra $\mathfrak{g}$ of G. The universal enveloping algebra $\mathfrak{u}(\mathfrak{g})$ of $\mathfrak{g}_C$ acts on $C^\infty(G)$. The left (resp. right) action of b in $\mathfrak{u}(\mathfrak{g})$ will be denoted by bf (resp. fb) for f in $C^\infty(G)$. Let $\mathfrak{z}$ be the center of $\mathfrak{u}(\mathfrak{g})$ and $d(p, q)$ the Riemannian distance on the symmetric space G/K where K is a maximal compact subgroup of G. Define a function d on G and a seminorm $\| \ \|_p$ on $C^\infty(G)$ by

$$d(x)=d(xo, o), \ o \text{ is the origin in } G/K$$

and

$$\|f\|_p^2=\lim_{\varepsilon\to+0} \varepsilon^p \int_G |f(x)|^2 e^{-\varepsilon d(x)} dx \quad \text{for } f \text{ in } C^\infty(G)$$

where p is a nonnegative real number and dx is the Haar measure on G.

Definition I. Let χ be a character of $\mathfrak{z}$. The space $H_p(G, \chi)$ is defined as the set of all C^∞-functions f satisfying $\|b_1 f b_2\|_p<\infty$ and $(z-\chi(z))f=0$ for all b_i in $\mathfrak{u}(\mathfrak{g})$ and z in $\mathfrak{z}$. $H_p(G, \chi)$ is a topological G-module with the canonical actions. Furthermore $\|R_x f\|_p=\|L_x f\|_p=\|f\|_p$ for x in G and f in $H_p(G, \chi)$ where R and L are respectively the right and left actions of G on $H_p(G, \chi)$. Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$. We denote the root space decomposition of $\mathfrak{g}_C$ by $\mathfrak{g}_C=\mathfrak{h}_C\oplus\sum_{\alpha\in\Phi}\mathfrak{g}_\alpha$ where Φ is the root system of $(\mathfrak{g}_C, \mathfrak{h}_C)$. Select, for each α in Φ, X_α in $\mathfrak{g}_\alpha$ satisfying $B(X_\alpha, X_{-\alpha})=1$

Received January 21, 1987.