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Irreducibility of Discrete Series Representations for Semisimple Symmetric Spaces

David A. Vogan, Jr.*

§ 1. Introduction

Let G be a connected reductive Lie group and H a symmetric subgroup. This means that there is an involution (an automorphism of order two) σ of G with the following properties: σ is trivial on H, and H contains the identity component of the fixed point set of σ. The quotient space G/H is a typical *reductive symmetric space*. (We allow G to be reductive instead of only semisimple to facilitate inductive arguments). Such a homogeneous space carries a G-invariant measure, so there is a unitary representation of G on $L^2(G/H)$. The representations of G on irreducible subrepresentations of G on $L^2(G/H)$ are called the *discrete series representations of* G *on* G/H. Write $L^2(G/h)_d$ for the sum of all these discrete series.

Building on work of Flensted-Jensen, Oshima and Matsuki in [Oshima-Matsuki] (1984) have given a detailed description of all discrete series representations of G/H. There is a parameter set $\mathscr{P}$ (roughly the characters of a certain compact torus satisfying some regularity and evenness conditions). For each X in $\mathscr{P}$, they construct a unitary representation $A(X)$ and an embedding of $A(X)$ in $L^2(G/H)_d$. Then they prove that

$$L^2(G/H)_d = \bigoplus_{X \in \mathscr{P}} A(X) \tag{1.1}$$

(What Flensted-Jensen did was to construct $A(X)$ and the embedding for "most" X.)

Our concern in this paper is with a small technical question: whether the representation $A(X)$ are irreducible. The most interesting question of this nature is a weaker one: whether (1.1) diagonalizes the invariant differential operators on G/H. That much is clear from the work of Oshima and Matsuki. In fact their proof is so compelling that (1.1) is clearly the "right" decomposition in some sense. Nevertheless, the irre-

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