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Characteristic Varieties of Highest Weight Modules and Primitive Quotients

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Dedicated to Professor Ichiro Satake on his 60th birthday

§ 0. Introduction

0.1. We discuss about the characteristic varieties of certain modules over the enveloping algebra of a semisimple Lie algebra, such as highest weight modules and primitive quotients.

Let G be a connected semisimple algebraic group over the complex number field $\boldsymbol{C}$, $\mathfrak{g}$ its Lie algebra and $U(\mathfrak{g})$ the enveloping algebra of $\mathfrak{g}$. Let X be the flag variety of G and $\mathscr{D}_X$ the sheaf of linear (algebraic) differential operators on X. The natural action of G on X induces an algebra homomorphism $U(\mathfrak{g}) \to \Gamma(X, \mathscr{D}_X)$. Hence for a $U(\mathfrak{g})$-module M we have a $\mathscr{D}_X$-module $\mathscr{D}_X \otimes_{U(\mathfrak{g})} M$.

For a finitely generated $U(\mathfrak{g})$-module M (resp. a coherent $\mathscr{D}_X$-module $\mathscr{M}$) the associated variety $V(M)$ (resp. the characteristic variety $\mathrm{Ch}(\mathscr{M})$) is a subvariety of the dual space $\mathfrak{g}^*$ of $\mathfrak{g}$ (resp. a subvariety of the cotangent bundle T^*X). For simplicity we sometimes write $\mathrm{Ch}(M)$ instead of $\mathrm{Ch}(\mathscr{D}_X \otimes_{U(\mathfrak{g})} M)$ for a finitely generated $U(\mathfrak{g})$-module M and call it the characteristic variety of M.

We hope to determine the associated varieties and the characteristic varieties of the irreducible highest weight modules L with trivial central character and the quotients $U(\mathfrak{g})/I$, where I is a primitive ideal of $U(\mathfrak{g})$ with trivial central character. For a finitely generated $U(\mathfrak{g})$-module M with trivial central character $V(M)$ is determined from $\mathrm{Ch}(M)$ (Borho-Brylinski, see Proposition 1.2 below). Hence our problems are the following:

Problem 0.1. *Determine* $\mathrm{Ch}(L)$ *for irreducible highest weight modules* L *with trivial central character.*

Problem 0.2. *Determine* $\mathrm{Ch}(U(\mathfrak{g})/I)$ *for primitive ideals* I *of* $U(\mathfrak{g})$

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