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The Space of Eisenstein Series in the Case of GL_2

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Introduction

It is known in the classical cases and also expected to be true in general that every automorphic form orthogonal to cusp forms is a linear combination of Eisenstein series. Among the classical and recent references are Hecke [6], Kloosterman [8], Gundlach [4], Maass [11], Roelcke [13], Shimizu [14], Shimura [15]. [6], [8], [4] and [14] treat holomorphic cases, while [11] and [13] treat real analytic cases. [15] proves the most general results known so far for Hilbert modular groups (it discusses also the case of half-integral weights).

In this note we consider the group GL_2 over an arbitrary number field, to show that the assertion in the biginning is valid for automorphic forms on that group which are eigenfunctions of bi-invariant differential operators; here we understand that 'a linear combination' of Eisenstein series includes a process of taking derivatives or residues with respect to a parameter.

We do not try to make our exposition self-contained. In fact, the automorphic representation theory and the fundamental property of Eisenstein series (analytic continuation etc.) are assumed. As to the first subject the basic reference is Jacquet-Langlands [7]. As to the second subject there are many references: Langlands [10], Harish-Chandra [5], Kubota [9], Gelbart-Jacquet [3], Arthur [1], Shimura [15].

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§ 1. Automorphic forms

1. Throughout this note F denotes an algebraic number field of finite degree. Let G be the group GL_2 viewed as an algebraic group over F so that $G_F = GL_2(F)$. Let P be the set of all places of F and P_f (resp. P_∞) the set of all finite (resp. infinite) places in P. For $v \in P$ we write

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