Advanced Studies in Pure Mathematics 13, 1988
Investigations in Number Theory
pp. 479–492

On Generalized Periods of Cusp Forms

Katsumi Shiratani

Introduction

Manin [1], [2] defined a p-adic measure and p-adic Hecke series attached to cusp forms with respect to the full modular group.

In the present paper it is our aim to give a few remarks on p-adic measures attached to the Bernoulli functions and the cusp forms, and to give a p-adic expression of generalized periods of the cusp forms. Namely we here discuss the generalized periods of the cusp forms with any Dirichlet characters.

§ 1. Nasybullin's lemma

We set $q=p$ for any prime $p>2$ and $q=4$ for the prime $p=2$. Let $\bar{f}=[f, q]$ be the least common multiple f and q, and $\boldsymbol{Z}$ the rational integer ring.

The ring $\boldsymbol{Z}_{\bar{f}}=\varprojlim_{n} \boldsymbol{Z}/p^n\bar{f}\boldsymbol{Z}$, $n\geqq 0$, the inverse limit with natural homomorphisms, is isomorphic to the direct product of the rational p-adic integer ring $\boldsymbol{Z}_p$ and the residue class ring $\boldsymbol{Z}/f_0\boldsymbol{Z}$ with a natural number f_0 such that $\bar{f}=p^l f_0$, $(f_0, p)=1$.

Let $\boldsymbol{Z}_{\bar{f}}^*$ be the multiplicative group of $\boldsymbol{Z}_{\bar{f}}$, so that it is isomorphic to the direct product of the unit groups $\boldsymbol{Z}_p^*$ and $(\boldsymbol{Z}/f_0\boldsymbol{Z})^*$.

Let K be a field over the rational p-adic number field $\boldsymbol{Q}_p$. Then we call a function μ a K-measure on $\boldsymbol{Z}_{\bar{f}}^*$, if μ is a finitely additive function defined on open-closed subsets in $\boldsymbol{Z}_{\bar{f}}^*$, whose values are in the field K. Any open-closed subset in $\boldsymbol{Z}_{\bar{f}}^*$ is a disjoint union of some finite intervals $I_{a,n}=a+p^n\bar{f}\boldsymbol{Z}_{\bar{f}}$ in $\boldsymbol{Z}_{\bar{f}}^*$, where $a\in \boldsymbol{Z}$ prime to $\bar{f}$, and therefore a K-measure μ is determined by its values on all the intervals in $\boldsymbol{Z}_{\bar{f}}^*$.

Let $\boldsymbol{Q}^{(f)}$ denote the set of such rational numbers, each denominator of which is a divisor of $\bar{f}p^n$ for some $n\geqq 0$.

Then Nasybullin's lemma reads as follows [1].

Lemma 1. *Let R be a K-valued function defined on $\boldsymbol{Q}^{(f)}$ with a property: There exist two constants A, $B\in K$ such that*

Received December 17, 1986.