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# A Table for Pure Cubic Fields

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Let $K$ be a cubic number field with discriminant $D<0$. In the previous paper [3], we have utilized an elliptic unit for the simultaneous determination of a fundamental unit $e$ and the class number $h$ of $K$. The significance of the method is that no calculation in $K$ is necessary; exactly, one can determine $e$ and $h$ by some arithmetic of an imaginary quadratic field $Q(\sqrt{D})$ and by approximate evaluation of the Dedekind eta function. As a process of the algorithm is fully explained in [3] with numerical examples, we do not repeat it here.

For pure cubic fields, our method has been implemented as a FORTRAN program on HITAC M-280H, Computer Center, University of Tokyo. It is arranged to apply to any pure cubic field $K$ with $300\leq |D|\leq 4800000000$ provided that C.P.U. time is not restricted. As a preparation, a primitive ring root modulo $p$ defined in [3] was computed for any prime number $p<40000$ beforehand, which took 327 seconds of total C.P.U. time. Further, the constants $\pi$ and $\pi\sqrt{-3}$ were computed to 40000 decimal digits precision in advance, which took 1766 seconds of C. P. U. time.

By means of the program, we have actually computed $e$ and $h$ for 358 pure cubic fields with $300\leq|D|\leq 1153200$, which took 25758 seconds of total C.P.U. time, though 1350 seconds of C.P.U. time was required for 157 fields with $300\leq|D|\leq 270000$. It should be mentioned that there are several numerical results for the class number $h$ as in [2] and others. But, on the other hand, there exist quite a few numerical results for the fundamental unit $e$, not the regulator, only as in [1] and [5]. Therefore, it will be useful to present precise $e$ in a table. By calling the program deposited in Computer Center, University of Tokyo, anyone can get extended data.

Now, let us describe the table below which has appeared in [4]. Every pure cubic field $K$ is represented by a unique pair $(a, b)$ of coprime square free natural numbers $a, b, a>b$, such that the defining equation of $K$ is given by $X^3-ab^2=0$. Then $D=-3f^2$, where $f=ab$ or $3ab$ respectively when $a^2-b^2$ is or is not divisible by 9. Starting from $f=10$, namely excluding the cases $(a, b)=(2, 1), (3, 1)$, the table is given in increasing order

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