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Selberg Trace Formula for a Certain Group Generated by $PSL(2, Z[i])$ and a Reflection

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Introduction

Let H be the three dimensional upper half space consisting of all elements $u=(z, v)$, where z is a complex number and v is a positive real number. Moreover, put $G=PSL(2, C)$ and $\Gamma=PSL(2, Z[i])$. It is well known that Selberg trace formula for the Hilbert space $L^2(\Gamma\backslash H)$ holds, which has already been presented by various methods in [6], [7] or [8] for example. Let now ω: $(z, v)\to(\bar{z}, v)$ be the complex conjugation with respect to the z-part of $u=(z, v)$. Then the space $L^2(\Gamma\backslash H)$ has the direct sum decomposition $L^2(\Gamma\backslash H)=V_e\oplus V_o$ in accordance with the operation of ω, where V_e and V_o are spaces defined respectively by $V_e=\{f\in L^2(\Gamma\backslash H)\mid f(\omega u)=f(u)\}$ and $V_o=\{f\in L^2(\Gamma\backslash H)\mid f(\omega u)=-f(u)\}$.

The purpose of the present paper is to derive trace formulas for V_e and V_o in explicit forms. Let $\tilde{\Gamma}=\langle\Gamma, \omega\rangle$ be the group generated by Γ and ω. Then, the equality $L^2(\tilde{\Gamma}\backslash H)=V_e$ is shown to hold from the definition of them. Hence, it is sufficient to consider the trace formula for the space $L^2(\tilde{\Gamma}\backslash H)$ by means of Selberg's theory to obtain that for V_e. The trace formula for V_o then follows easily from those for $L^2(\Gamma\backslash H)$ and $V_e(=L^2(\tilde{\Gamma}\backslash H))$.

On the other hand, in [10], we investigated the trace formula for the Hilbert space with the group generated by $PSL(2, Z)$ and the reflection with respect to the imaginary axis. Though the material in this article looks analogous to that of [10], the actual analysis involved seems to be rather different. In fact, in the present case, only relatively parabolic elements defined in Section 2 contribute the continuous spectrum, and relatively identical elements in Section 2, which exist clearly, do not contribute the continuous spectrum, while, in [10], the elements, which correspond to our relatively identical elements in the classification in Section 2, contribute the continuous spectrum, and no elements correspond to our relatively parabolic elements.

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