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On a Classical Theta-Function, II

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The present paper, containing a partial and expository reconstruction of the results which were known since [3], is written for the purpose of stating some basic facts on a classical theta-function in a form which is possibly convenient in investigations related positively to metaplectic groups.

Since this paper is a continuation of [2], the ordinals of all sections, theorems, propositions and formulas follow those of [2], while references and footnotes are numbered anew, and the only theorem in [2] is quoted as Theorem 1.

§ 3. Eisenstein series $E(z, s)$

Having finished the investigation of the automorphic factors of the theta function (1), we are naturally led to the following Eisenstein series:

$$E(z, s)=\sum_{\Gamma_0\backslash\Gamma} \chi(\sigma, 1)e^{-(1/2)i \arg (cz+d)}\frac{y^{s/2}}{|cz+d|^s}. \tag{14}$$

Here, z is a point in the upper half plane H, s is a complex number, Γ_0 is the group consisting of all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$ with $c=0$, and $\chi(\sigma, 1)$ is as in Theorem 1. Moreover, $\arg(cz+d)$ is always normalized by

$$-\pi \leqq \arg (cz+d) < \pi \tag{15}$$

in accordance with (2). The series (14) is absolutely convergent for $\mathrm{Re}\, s>2$, and satisfies the transformation formula

$$E(z, s)=\chi(\sigma, 1)e^{-(1/2)i \arg (cz+d)}E(\sigma z, s), \qquad (\sigma \in \Gamma). \tag{16}$$

Therefore one can expect that $E(z, s)$ may coincide with $\vartheta(z)$ at $s=\frac{1}{2}$. That this is actually the case will be shown in Section 7.

In this section, we shall observe the effect on $E(z, s)$ of the invariant differential operator

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