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A_4-extensions over Real Quadratic Fields and Hecke Operators

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§ 0. Introduction

In articles [10], [11], Shimura investigated the relation between the arithmetic of real quadratic fields and cusp forms of real "Neben"-type of weight 2. He showed that the eigenvalues of Hecke operators for such forms are closely connected with the reciprocity law in certain abelian extensions of a real quadratic field k and, moreover, such extensions can be generated by the coordinates of certain points of finite order on an abelian variety associated with these cusp forms. Later, his results were enriched by several authors Doi-Yamauchi [2], Ohta [8] and Koike [4]. Especially, in [4], we understood his result through congruences between the cusp forms of weight 2 and cusp forms of weight 1 which are obtained from Mellin transform of L-functions of the real quadratic field k. These cusp forms of weight 1 correspond to dihedral representations of the Galois group G_Q.

In this paper, we investigate several examples of cusp forms of real "Neben"-type of weight 2 which are congruent to cusp forms of weight 1 corresponding to representations of the Galois group G_Q of type S_4. We also discuss arithmetic properties analogous to the above Shimura's result induced from these congruences.

To state our result precisely, we introduce several notations. Let p, $p\equiv 1 \pmod 4$ be a prime. Let $f(z)=\sum_{n=1}^{\infty} a_n q^n$, $a_1=1$, $q=e^{2\pi\sqrt{-1}z}$, be a primitive form in $S_2\left(p, \left(\frac{}{p}\right)\right)$ where $\left(\frac{}{p}\right)$ denotes the Legendre symbol. Put $K_f=Q(a_n|n\geqq 1)$ the coefficient field of the cusp form $f(z)$. Then K_f is an imaginary CM-field. Let F_f denote the maximal real subfield of K_f. We denote by $\mathfrak{o}_K$ (resp. $\mathfrak{o}_F$) the ring of integers in K_f (resp. F_f). Put $2d=[K_f: Q]$. We fix a prime divisor $\tilde{\mathfrak{p}}$ of the algebraic closure $\bar{Q}$ of Q lying over p. Let ρ denote the complex conjugation.

Prime ideals which Shimura considered in [10] [11] are ramified in the relative quadratic extension K_f over F_f, and they are closely related to the

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