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A Tripling Symbol for Central Extensions of Algebraic Number Fields

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Let K/k be a finite abelian extension of a finite algebraic number field and M be a Galois extension of k which contains K. Denote by $\hat{K}_{M/k}$ and $K^*_{M/k}$ the maximal central extension of K/k in M and the genus field of K/k in M. Since K/k is abelian, $K^*_{M/k}$ coincides with the maximal abelian extension of k in M. In general, the Galois group $G(\hat{K}_{M/k}/K^*_{M/k})$ is isomorphic to a quotient group of the dual $M(G)=H^{-3}(G, Z)$ of the Schur multiplier $H^2(G, Q/Z)$ of G. If M is enough large, $G(\hat{K}_{M/k}/K^*_{M/k})$ is isomorphic to $M(G)$. In such a case, we call M abundant for K/k.

Furuta [2] gives a prime decomposition symbol $[d_1, d_2, p]$ which indicates the decomposition in $\hat{K}_{M/k}/K^*_{M/k}$ of a prime p which is degree 1 in $K^*_{M/k}$, where $k=Q$, $K=Q(\sqrt{d_1}, \sqrt{d_2})$ and M is a ray class field of K which is abundant for K/k. Also it proves the inversion formula $[p_1, p_2, p_3]=[p_1, p_3, p_2]$ except only a case.

Akagawa [1] extended this symbol to $(x, y, z)_n$ for any kummerian bicyclic extension $K=k(\sqrt[n]{x}, \sqrt[n]{y})$ over any base field k with serveral conditions which make $(x, y, z)_n$ and $(x, z, y)_n$ defined and the inversion formula $(x, y, z)_n(x, z, y)_n=1$ be true. This contains the proof of the excepted case of Furuta [2].

In this paper, we extend the symbol [, ,] as a character of the number knot modulo $\mathfrak{m}$ of K/k with $\mathfrak{m}$ being a Scholz conductor of K/k which is defined in Heider [4]. The character is defined by using the inverse map $H^{-1}(G, C_K)\cong H^{-3}(G, Z)$ (of Tate's isomorphism), which is obtained by translating the norm residue map of Furuta [3], which is written in ideal theoretic, into idele theoretic. In our definition, the extension K/k may be any bicyclic extension $K=k_{\chi_1}\cdot k_{\chi_2}$ with χ_1, χ_2 being global characters. But the symbol is of type (χ_1, χ_2, c), where c is contained in the number knot. So we can consider the inversion formula only in the case when χ_1 and χ_2 are Kummer characters $\chi_a^{(n)}$ and $\chi_b^{(n)}$. When that is the case, we put $(a, b, c)_n=(\chi_a^{(n)}, \chi_b^{(n)}, c)$ and calculate $(a, b, c)_n+(a, c, b)_n$ (which are written additively in this paper). We approach this result to a necessary and sufficient condition of the inversion formula $(a, b, c)_n+(a, c, b)_n=0$, by

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