Advanced Studies in Pure Mathematics 12, 1987
Galois Representations and Arithmetic Algebraic Geometry
pp. 343–373

Class Field Theory for Two Dimensional Local Rings

Shuji Saito

Contents

Introduction
Notations
§1 The statements of the main results
§2 The determination of Ker (Ψ_K)
§3 The ramification along the special fiber
§4 A finiteness theorem
§5 The Hasse principle for A
§6 A duality theorem on the p-primary part
§7 The proof of the existence theorem (the prime-to-p part)
§8 The proof of the existence theorem (the p-primary part)
References

Introduction

Let A be an excellent normal two-dimensional henselian local ring with finite residue field F and quotient field K. The purpose of this paper is to construct the class field theory for K, using the method and results in [S–1]. Let P be the set of all prime ideals of height one in A. For each $\mathfrak{p} \in P$, let $A_\mathfrak{p}$ be the henselization of A at $\mathfrak{p}$ and let $K_\mathfrak{p}$ (resp. $\kappa(\mathfrak{p})$) be its quotient (resp. residue) field. Then, $K_\mathfrak{p}$ is a henselian two-dimensional local field in the sense of [K–1] (cf. also [S–1] (2.2)). For such a field, K. Kato constructed the class field theory in [K–1] and [K–2]. Then, our method is to put together these local theories, which is a standard technique in the classical class field theory. To state our main results, we recall briefly some results in [K–1] and [K–2]: In general, for a noetherian scheme Z, Put $H^1(Z)=H^1(Z_{\mathrm{et}}, \boldsymbol{Q}/\boldsymbol{Z})$ which is identified with the Pontrijagin dual of the abelian fundamental group $\pi_1^{\mathrm{ab}}(Z)$. For a noetherian ring R, we put $H^1(R)=H^1(\mathrm{Spec}\,(R))$. For each $\mathfrak{p} \in P$, Kato constructed a canonical homomorphism

$$\Psi_{K_\mathfrak{p}}\colon H^1(K_\mathfrak{p}) \longrightarrow (K_2(K_\mathfrak{p}))^*_{\mathrm{tor}},$$

Received April 2, 1986.