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Wild Ramification in the Imperfect Residue Field Case

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Introduction

The aim of this paper is to study wild ramification of complete discrete valuation fields without the assumption that the residue field is perfect. In the case where the residue field is perfect, there is a beautiful theory of ramification groups (Serre [14] §4). The difficulty of our case is that there seems to be no such a theory in general. As applications of our study, we shall show the following three results. In the following, let K be a complete discrete valuation field of mixed characteristics $(0, p)$.

(1) Miki [9], [10] studied $\boldsymbol{Z}_p$-extensions of K. He showed that any $\boldsymbol{Z}_p$-extension of K is contained in a composite of an unramified extension of K and a $\boldsymbol{Z}_p$-extension of the "canonical subfield" k of K. Here, k is characterized by the following properties.

(0–1–1) k is complete with respect to the valuation induced from K.

(0–1–2) The residue field of k is the maximal perfect subfield of the residue field $\bar{K}$ of K.

(0–1–3) k is algebraically closed in K.

We generalize his result by using continuous cohomology (Tate [16]). Let $H^q(G, A)$ be the q-th continuous cohomology group of a topological group G with coefficients in a topological G-module A. Let $G_E = \mathrm{Gal}(E_{\mathrm{sep}}/E)$ be the absolute Galois group of a field E, and let (r) denote the r-th Tate twist for $r \in \boldsymbol{Z}$ (cf. Tate [16] p. 262).

Theorem (0–2). *Assume that the residue field $\bar{K}$ of K is separably closed. Then the inflation map induces an isomorphism*

$$H^1(G_k, \boldsymbol{Z}_p(r)) \xrightarrow{\simeq} H^1(G_K, \boldsymbol{Z}_p(r)) \quad \textit{if} \quad r \neq 1.$$

As $H^1(G_E, \boldsymbol{Z}_p) \simeq \mathrm{Hom}(G_E, \boldsymbol{Z}_p)$ classifies $\boldsymbol{Z}_p$-extensions of a field E, Miki's result is the particular case $r=0$ of Theorem (0–2). (Miki's result can be reduced to the case where $\bar{K}$ is separably closed).

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