Advanced Studies in Pure Mathematics 12, 1987
Galois Representations and Arithmetic Algebraic Geometry
pp. 221–234

On a Question Arising from Complex Multiplication Theory

Greg W. Anderson

§ 0. Introduction

An abelian variety A defined over $\boldsymbol{C}$, equipped with complex multiplication and level structure, is described up to isomorphism by some invariants that are "analytic" in nature. (The details of this description are reviewed in Section 1.) Let s be an arbitrary automorphism of $\boldsymbol{C}$. Tate conjectured [8] and Deligne proved [1] a formula for the analytic invariants of A^s, the conjugate of A under s, in terms of classfield theory. (See also Lang [4], in which summaries of the contents of [1, 8] can be found.) Tate's formula generalizes the classical reciprocity law of Shimura-Taniyama [6, 7] to the case in which s does not necessarily fix the CM type of A. (Tate's formula is reviewed in Section 1.)

Now figuring prominently in Tate's formula is a certain cocycle. The task we set for ourselves in this paper is to abstract the construction of the cocycle figuring in Tate's formula making possible the subsequent specialization of that construction to the function field case. This task is carried out in Section 2.

The eponymous question of the paper is not the question *answered* by the investigation of Section 2, but rather the question *raised* by it: What interpretation can be given to the new cocycle which we have constructed in the function field case? This is an open problem; the author expects the solution to be found in an as-of-yet-undeveloped theory of higher-dimensional Drinfeld modules with complex multiplication in which, in particular, an analogue of Tate's formula is valid.

Acknowledgement. The author gratefully acknowledges the support and hospitality of the RIMS in Kyoto during the author's visit in October 1985.

§ 1. A basic problem of complex multiplication theory

1.0. Notation. We denote by $\overline{\boldsymbol{Q}}$ the algebraic closure of $\boldsymbol{Q}$ in $\boldsymbol{C}$.

Received December 20, 1985.