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Some Problems on Three Point Ramifications and Associated Large Galois Representations

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Dedicated to Professor Ichiro Satake for his 60th birthday

Introduction

Let $\boldsymbol{Q}$ be the rational number field, $\bar{\boldsymbol{Q}}$ be its algebraic closure, and l be a fixed prime number. Then the absolute Galois group $G_{\boldsymbol{Q}}=\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ admits a canonical representation

$$\varphi=\varphi_{\boldsymbol{Q}}\colon G_{\boldsymbol{Q}}\longrightarrow \mathrm{Out}\,\pi_1^{\text{pro-}l}(\boldsymbol{P}^1_{\bar{\boldsymbol{Q}}}\backslash\{0,1,\infty\}),$$

in the outer automorphism group of the pro-l fundamental group of the punctured projective line, which arises from the exact sequence

$$1\longrightarrow\pi_1^{\text{pro-}l}(\boldsymbol{P}^1_{\bar{\boldsymbol{Q}}}\backslash\{0,1,\infty\})\longrightarrow\pi_1^{\text{pro-}l}(\boldsymbol{P}^1_{\boldsymbol{Q}}\backslash\{0,1,\infty\})\longrightarrow G_{\boldsymbol{Q}}\longrightarrow 1.$$

Recently, several authors started (perhaps more or less independently) to work on this type of "large Galois representations"; Belyĭ [3], Grothendieck [7], Deligne [5], [6], the author [9], [10], etc. In this report, we pose and discuss various basic open problems related to this represetation φ and its natural "subrepresentations" ψ.

§ 1. The Galois representation φ

(1–1) First, let us repeat the definition of the Galois representation $\varphi_{\boldsymbol{Q}}$ more precisely in terms of function fields. Let M be the maximum pro-l extension of the rational function field $K=\bar{\boldsymbol{Q}}(t)$ unramified outside $t=0, 1, \infty$. Then $M/\boldsymbol{Q}(t)$ is also a Galois extension. So, identifying the two Galois groups $\mathrm{Gal}(K/\boldsymbol{Q}(t))$ and $G_{\boldsymbol{Q}}=\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ in the obvious way, we obtain an exact sequence of Galois groups

$$1\longrightarrow \mathrm{Gal}(M/K)\longrightarrow \mathrm{Gal}(M/\boldsymbol{Q}(t))\longrightarrow G_{\boldsymbol{Q}}\longrightarrow 1.$$

Put $\mathfrak{F}=\mathrm{Gal}(M/K)$ and $\tilde{\mathfrak{F}}=\mathrm{Gal}(M/\boldsymbol{Q}(t))$. Then the composite of three canonical homomorphisms

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