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Linear Representations of the Galois Group over Local Fields

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Let F be a p-field, i.e. a complete discrete valuation field with a finite residue field of characteristic p, F^{alg} be an algebraic closure of F, F^{sep} be the separable closure of F in F^{alg}. Let $G = \mathrm{Gal}(F^{\mathrm{alg}}|F) = \mathrm{Gal}(F^{\mathrm{sep}}|F)$. It is a profinite group with the Krull topology. For a profinite group H, let $R(H)$ (resp. $\hat{H}$) denote the set of the equivalence classes of the finite dimensional continuous (resp. irreducible) representations σ over the complex number field. We are concerned with the classification or the parametrization of the set $\hat{G}$.

0.1. In [4], we gave a parametrization of a certain family of irreducible supercuspidal representations of $GL_n(F)$ induced from a certain class of representations (= très cuspidal representations of Carayol [1]) in terms of multiplicatively generic elements in F^{sep} (cf. § 5). In view of local Langlands conjecture (cf. [8]), it is desirable to have a similar parametrization for $\hat{G}$.

0.2. There are many interesting results on Galois representations, only a few of which I had occasions to study carefully. Primitive representations are studied by Weil [11], and finally classified by Koch [5]. Tame representations (i.e. $(\deg\sigma, p) = 1$) are classified by Howe, Koch-Zink [7] or Moy [9]. Representations of degree p are classified by Koch [6]. However, the method employed there does not seem to match the way I imagine. In my talk at the meeting, I reported partial results on the parametrization, and more generally on the study of Galois representations based on the natural filtration of G by the absolute (upper) ramification groups G^v. But, later I noticed that, in the latter respect, a substantial work had been done by Deligne-Henniart [2]. Therefore in this note, confining ourselves to the former respect, we give a criterion of irreducibility of induced representations in Section 2, some reduction by r-invariant in Section 3, the description of the dual group of G^v/G^{v+} in Section 4, and its relation to

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