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On Some Properties of the Universal Power Series for Jacobi Sums

Yasutaka Ihara, Masanobu Kaneko and Atsushi Yukinari

Introduction

The subject of this paper is the l-adic power series $F_\rho(u, v)$ in two variables associated to each element ρ of the absolute Galois group Gal $(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ over the rationals, constructed and studied in [PGC]. This was constructed by using pro-l étale coverings of $\boldsymbol{P}^1\backslash\{0, 1, \infty\}$, and was shown to be "universal" for Jacobi sums of l-power exponents. This study was then taken up, also by G. Anderson and R. Coleman, and we now have a deeper understanding of F_ρ. We shall prove two theorems on F_ρ, one on a (non-obvious) functional equation (Theorem A, § 1), and the other, on the determination of the coefficients (Theorem B, § 1) when $\rho \in \mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q}(\boldsymbol{\mu}_{l^\infty}))$. Since these results were also obtained by Anderson and Coleman (see below for details), the stress will be laid on the difference of methods in proofs.

To be more precise, denote by Ω_l the maximum abelian pro-l extension of the cyclotomic field $\boldsymbol{Q}(\boldsymbol{\mu}_{l^\infty})$ unramified outside l, and by Ω_l^{ur} the maximum everywhere unramified subextension of $\Omega_l/\boldsymbol{Q}(\boldsymbol{\mu}_{l^\infty})$;

$$\boldsymbol{Q}\subset\boldsymbol{Q}(\boldsymbol{\mu}_{l^\infty})\subset\Omega_l^{\mathrm{ur}}\subset\Omega_l.$$

Put

$$\mathfrak{g}_0=\mathrm{Gal}\,(\Omega_l/\boldsymbol{Q})\supset\mathfrak{g}_1=\mathrm{Gal}\,(\Omega_l/\boldsymbol{Q}(\boldsymbol{\mu}_{l^\infty}))\supset\mathfrak{g}_2=\mathrm{Gal}\,(\Omega_l/\Omega_l^{\mathrm{ur}}).$$

Then the association $\rho\to F_\rho$ factors through a 1-cocycle

$$\text{(1)}\qquad \mathfrak{g}_0\longrightarrow\mathscr{A}^\times,\qquad \mathscr{A}=\boldsymbol{Z}_l[[u, v]],$$

$\mathscr{A}^\times$ being regarded as a module over $\boldsymbol{Z}_l^\times=\mathfrak{g}_0/\mathfrak{g}_1$ by

$$\text{(2)}\qquad j_\alpha\colon 1+u\longrightarrow(1+u)^\alpha,\ 1+v\longrightarrow(1+v)^\alpha\qquad(\alpha\in\boldsymbol{Z}_l^\times).$$

As shown in [PGC], the *special values* $F_\rho(\zeta-1, \zeta'-1)$ ($\zeta, \zeta'\in\boldsymbol{\mu}_{l^n}$, ρ: a

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